Applications

Simulation

Simulation: Transactions of the Society for Modeling and Simulation International
1–21

DOI: 10.1177/00375497221132574
journals.sagepub.com/home/sim
SAGE SCS

Multi-Objective Patient Appointment Scheduling Framework (MO-PASS): a data-table input simulation–optimization approach

Mohammad Dehghanimohammadabadi[1], Mandana Rezaeiahari[2] and Javad Seif[3]

Abstract

Appointment scheduling is one of the critical factors for improving patient satisfaction with healthcare services. A practical and robust appointment scheduling solution allows clinics to efficiently utilize medical devices, equipment, and other resources. This study introduces a Multi-Objective Patient Appointment Scheduling (MO-PASS) framework to enhance clinic operations and quality of care. The proposed framework integrates three modules: (1) Optimization (using MATLAB), (2) Data-Exchange (MS Excel), and (3) Simulation (Simio). To implement MO-PASS, the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm is coded in MATLAB, and a Simio API is developed, which exchanges simulated scenarios with MOPSO via Excel. The efficiency of the proposed framework is evaluated in a breast cancer clinic with multiple physicians and patient types. Two objective functions are defined for evaluating the solutions of the AS problem: (1) minimizing the total service time and (2) maximizing the number of (admitted) patients with zero overtime. Finally, the performance of MO-PASS is tested against three heuristic approaches with respect to objective functions. The computational experiment results show that the proposed MO-PASS outperforms the existing heuristic benchmarks. Also, the framework is accompanied by all the necessary details to make it practical and easy to implement.

Keywords

Multi-appointment scheduling, template scheduling, multi-objective scheduling, simulation-based optimization, simheuristics, Multi-Objective Particle Swarm Optimization (MOPSO)

1. Introduction

Efficient delivery of care is essential for every healthcare facility. However, due to high expenditures and ever-rising demands and patients' expectations, hospitals face an ongoing challenge to increase the efficiency of their operations.[1,2] One way to improve the quality of care is to develop a robust *patient appointment scheduling system* (PASS), which performs efficiently under various uncertainties. In addition, specialty care clinics mainly utilize these systems to manage access to service providers and hospitals to schedule patient appointments.[3] Therefore, an effective PASS is crucial to ensure patient satisfaction and effective healthcare services.

Appointment scheduling is defined as merely assigning specific start times for patients to receive care,[4] while, in practice, many other factors need to be considered to have realistic schedules. Some of these essential factors are listed below:

- *Planning Horizon.* Planning horizon in an appointment scheduling problem (ASP) can be considered at different levels, namely, *strategic*, *tactical*, and *operational*.[2] At the strategic level, decisions have long-term effects. Usually, they include the design of policies and capacity allocation, i.e., the policy of accepting walk-ins, the type of schedule, the

[1]Department of Mechanical & Industrial Engineering, Northeastern University, USA
[2]Fay W. Boozman College of Public Health, University of Arkansas for Medical Sciences, USA
[3]Department of Industrial and Manufacturing Engineering, California State Polytechnic University, Pomona, USA

Corresponding author:
Mohammad Dehghanimohammadabadi, Department of Mechanical & Industrial Engineering, Northeastern University, 360 Huntington Avenue, Boston, MA 02115, USA.
Email: m.dehghani@northeastern.edu

number of staff in each department, and more. At the tactical level, decisions are made for the medium-term horizon and may change yearly or seasonally. These decisions are usually related to allocating capacity to patient types, appointment intervals, appointment scheduling windows, etc. Finally, operational-level decisions aim to solve perennial problems.[5] These decisions are mainly bounded by strategic and tactical-level decisions[6] and are made daily. Decisions such as the allocation of patients to servers and appointment times and days[2] are some examples of operational-level decisions in ASP.

- *Appointment Types.* Depending on the care delivery procedure, a patient may require a single appointment (visit just a single provider), multiple appointments (visit different providers in different practice settings), or multi-stage appointments (visit multiple providers sequentially). The ASP becomes even more challenging when facing emergency and walk-in patients.
- *Complexity and Uncertainty.* Operational complexities of healthcare facilities are also critical in choosing an appropriate PASS. These complexities are impacted by some factors, such as stochastic service times, patients' unpunctuality, no-show rate, physicians' availability, and service interruptions.[7]
- *Multi-Objectives.* In ASP, multiple performance measures must be optimized simultaneously without a compromise. Depending on the healthcare settings, these objectives range from system performance measures such as resource utilization (i.e. physicians, nurses, rooms, etc.) and overtime constraints to patients' satisfaction criteria such as total waiting time or cycle time (i.e. length of stay). A well-designed PASS will likely improve a set of objectives independently without degrading others.

1.1. Motivation and objectives

A practical PASS model should consider the above-mentioned factors for a successful implementation. This paper aims to integrate simulation and optimization techniques as a new approach to solving ASP in outpatient clinics. The proposed approach determines the *allocation of capacity to patient types* (not individual patients). In other words, the final solution includes a *template of schedules* that indicates the allocated time slots for different patient types throughout the day for multiple physicians. Therefore, the proposed solution is considered a *tactical decision* subject to monthly or seasonal changes. Finally, this paper contributes to the ASP literature by applying a multi-objective approach to solve a stochastic version of ASP using simulation–optimization (SO). While SO approaches have been frequently used in ASP with single and weighted sum objectives, a few studies have applied SO to solve a multi-objective approach. Therefore, the main contributions of the paper are as follows:

- Defining a multi-objective ASP with multiple patient types and physicians;
- Designing, developing, and implementing a PASS using a novel SO approach;
- Applying the proposed model in a case study to demonstrate its efficiency compared with existing methods.

We demonstrate the application of the model through a case study, which is designed based on the operations found in a breast cancer clinic. This clinic serves multiple patient types under various process flows. Two competing objective functions are defined to solve this problem: (1) minimizing the total service time (TST) and (2) maximizing the number of (admitted) patients (NOP). Overtime is not allowed for the center, and a proper schedule must ensure all appointed patients are served by the end of the day. This model considers multi-stage appointments, in which every patient type receives a single appointment to enter the system and then follows its specific procedure to receive care from multiple servers. The number of servers is known for each stage, and all service times are stochastic.

The remainder of this paper is organized as follows: In section 2, relevant studies are reviewed to highlight the contribution of this work. The DSO model is introduced by [Dehghanimohammadabadi][8] which interested readers are encouraged to read the given reference to gain more insights The proposed framework is discussed in section 3, and its implementation details are provided in section 4. Section 5 demonstrated the applicability of the proposed model in a cancer clinic case study. Finally, concluding remarks and directions for future work are presented in section 6. For brevity, the proposed Multi-Objective Patient Appointment Scheduling System is defined as MO-PASS. For the readers' convenience, a list of abbreviations is provided in Appendix 1, and details of the mathematical problem formulation are shown in Appendix 2.

2. Literature review

In recent years, an increasing number of researchers have started to acknowledge the importance of patient ASPs due to its capability to facilitate the delivery of care in a complex healthcare environment. Existing methods in the literature are mostly restricted to optimization algorithms[7,9] or simulation modeling approaches.[10,11] Analytical methods such as queuing theory, stochastic programming, and

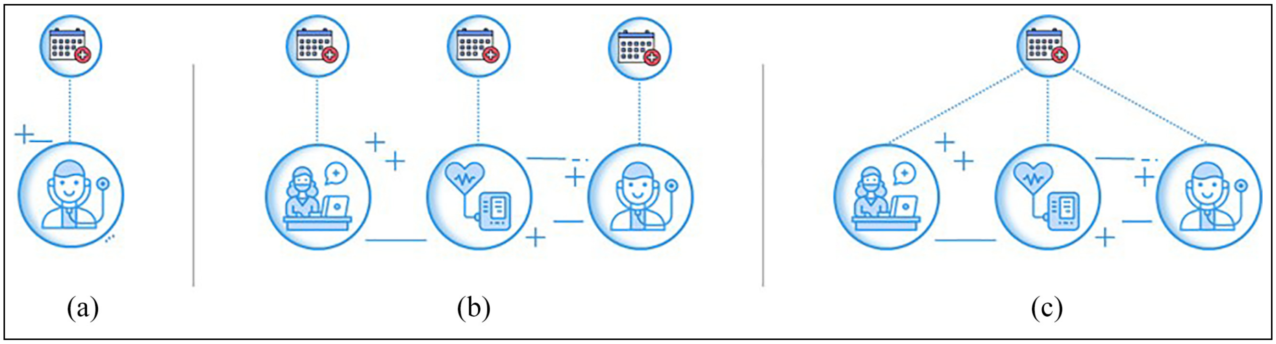


Figure 1. A visual representation of appointment types: (a) single-appointment scheduling, (b) multiple-appointment scheduling, and (c) multi-stage appointment scheduling.

non-linear programming are reasonable approaches for non-complex small-size problems with fundamental interactions between the model components.

This section provides an overview of related works in the literature to highlight the contribution of the presented study. To accomplish this, the existing papers are overviewed from two different perspectives, namely (1) *appointment types* and (2) *modeling approaches*. Since this work is focused on SO, the papers discussed in this section are mainly those that applied stochastic solution approaches to solve the problem.

2.1. Perspective 1: appointment types

Typically, there are three different problems in patients' apportionment scheduling. In the first type, (1) *single-appointment scheduling*, a patient receives a single appointment and only visits a single provider before leaving the healthcare facility. An example is a primary care or specialty care, where only doctor appointments are considered. The second type is called (2) *multiple-appointment scheduling*, in which multiple appointments are associated with the patient visiting different providers in different practice settings. An example of this case is rehabilitation departments, where patients need multiple specialists or resources from several departments. Finally, the third type is (3) *multi-stage appointment scheduling*, where multiple providers sequentially can serve the patient within one appointment slot. This type of appointment is primarily common when patients need diagnostic and undergo various tests on a single day. Figure 1 visually illustrates each of these appointment types. This section aims to provide an overview of studies with respect to the three types of appointment scheduling with the implication of SO approaches used within each category.

2.1.1. Multiple-appointment scheduling. Some multi-appointment scheduling papers model the problem as a flow shop, job shop, or open shop scheduling problem. Chien et al.[12] scheduled rehabilitation patients as a hybrid shop scheduling problem and considered partial precedence within the operations of physical therapy treatments. Jericä and Figueira[13] developed a multi-objective binary integer programming model for treatment scheduling and resource allocation of patients who should finish a specific treatment based on a pre-defined sequence. They assumed zero inter-waiting time between different procedures of the patients to optimize physicians' and equipment utilization and the number of treatments completed. Azadeh et al.[14] formulated a semi-online scheduling model for different types of patients at the pathology clinic. They modeled the problem as a hybrid shop scheduling problem with partial precedence constraints. Rezaeiahari and Khasawneh[15] used novel metaheuristic approaches to schedule patients with multi-appointments who visit providers sequentially. They assumed multiple providers for each patient type and partial providers' unavailability. Leeftink et al.[16] used a two-stage stochastic integer programming to optimize the template schedule for a multidisciplinary clinic with two patient types. In their study, walk-in patients are considered multidisciplinary cancer patients who are referred to as different clinicians after their cancer type is determined. This added a layer of complexity to their problem by making the daily demand of clinicians unknown.

2.1.2. Multi-stage appointment scheduling studies. In multi-stage scheduling papers, researchers often try to streamline the flow of patients on a set of pre-defined resources by including admission planning. Admission planning decides when to admit patients as a point of entry to the system to provide them with a sequence of care through multiple stages.[17] Therefore, a practical admission plan for multi-stage scheduling needs to consider various factors, such as patient flows, resources and physicians' availability, and the interaction between stages.

Albeit its importance, there has been little attention paid to the problem of multi-stage scheduling in the literature. Harper and Gamlin[18] used simulation to reduce the direct waiting time in a multi-stage outpatient clinic visit. White et al.[19] examined the relationship between capacity and patient flow in a multi-stage system to obtain the ideal number of exam rooms. Romero et al.[20] used a two-phase approach for capacity and admission planning of a dermatology outpatient clinic with multi-stage treatments. In their study, the authors used simulation to evaluate various appointment scenarios to schedule multidisciplinary patients with various treatment types. Results of this study show that the one-stop-shop appointment scenario, where the clinic has all the resources available for the diagnosis and treatment on the same day, would lead to less waiting time. Pérez et al.[21] formulated an offline and online scheduling model for patients with multi-step sequential procedures. In their model, they considered patients with stochastic arrival patterns and strict time-window constraints at the start of each step. Kuiper and Mandjes[22] used a tandem queue to balance patient waiting time versus the idle time of healthcare providers in a two-stage outpatient clinic. They determined various queuing-related quantities to find the best arrival epochs to minimize the objective function for patients with different types of service time distribution.

2.2. Perspective 2: modeling approaches

Uncertainty is an inevitable part of healthcare systems, which raises the need for a flexible and accurate solution approach. The ASP revolves around several uncertainty factors such as processing time, patient no-shows, patient unpunctuality, emergency patients, which all contribute to the complexity of the problem. Simulation modeling is a credible and flexible approach to studying complex and probabilistic systems such as healthcare operations as it can provide solutions to improve the system's performance. Studies conducted by Guo et al.[23] and Ogulata et al.[24] are examples of using simulation to determine appropriate appointment scheduling policies in healthcare settings. However, in the case of a large solution space, the simulation needs to be coupled with an optimization mechanism to explore the solution space efficiently and obtain the best configuration of the model in a timely manner.[25]

Considerable efforts have been made to develop new planning or PASS in recent years. Among these, simulation-based models received more attention from researchers and practitioners. Simulation is a powerful technique that embraces different complexities found in healthcare models to provide a desirable solution. Due to its capabilities to capture models' uncertainty and include non-linear interactions between operations,[9] simulation becomes a suitable tool to solve ASPs. Coupling simulation with optimization can make these capabilities even more effective. SO models are widely used in multidisciplinary scheduling problems[10] and have shown promising results.

SO approaches are becoming a promising avenue of research for stochastic complex models in general and ASPs in particular. This is because SO models can fully capture real-world problems' inherent complexity and uncertainty characteristics in detail without simplifying assumptions.

Recently, several studies have used a combination of simulation and optimization to address single-stage scheduling problems. Klassen and Yoogalingam[26] combined metaheuristic algorithms with simulation to determine the optimal appointment times for the patients at an outpatient clinic with a single provider. In their study, they propose a modification of a previously suggested dome-pattern. In another study, Klassen and Yoogalingam[27] showed that a combination of variable-length inter-arrivals and block scheduling outperforms the plateau dome rule for mitigating the effects of patient unpunctuality. Dehghanimohammadabadi et al.[28] used an SO approach to solve the problem of an outpatient clinic with two patient types and a single provider. Their goal was to find the best sequence of patients' arrival to minimize patients' average length of stay.

In addition to single-stage problems, SO has been applied in solving multi-stage appointment scheduling environments. Klassen and Yoogalingam[17] studied the benefit of mid-level service providers as an alternative for adding physician capacity in multi-stage outpatient clinics. In this work, the authors considered a variety of factors, including clinic size, patient unpunctuality, patients' allocation ratio to providers, and various scheduling rules to find the solution. They solved their model using OptQuest, a well-known SO add-in that comes with most commercial simulation packages. Saremi et al.[29] developed a multi-stage scheduling model with three stages, including pre-operation, surgery, and recovery. They evaluated the impact of the patient volume and service time variation at each stage. They used three simulation-based tabu search algorithms to schedule different types of patients and surgeons. In another study, Saremi et al.[30] optimized the scheduling of patients with heterogeneous service sequences in a multi-stage clinic using an SO approach. They tested the problem with different patient sizes and patient types within a horizon of a single day.

Baesler and Sepuä lveda[31] integrated simulation, goal programming, and genetic algorithms to schedule multiple procedures for medical oncology patients. Rezaeiahari and Khasawneh[32] used SO to schedule numerous procedures for domestic medical tourists traveling to destination medical centers. They examined the model's performance under different scenarios, including batch arrival and no-show probability.

In addition to the discrete event simulation (DES) approaches discussed above, agent-based simulation

(ABS) modeling is another practical approach to solving the ASP problem. Using ABS, one can evaluate changes in the infrastructure, patient behavior, and service design.[33] However, since the focus of this paper is not to model the human decision-making or agents' behavior, DES was deemed a more appropriate method than ABS. Interested readers can find some state-of-the-art articles in previous works.[34–37]

2.3. Contribution of this work

The studies mentioned above are listed in Table 1 to provide insights into their similarities and distinctions. As observed, there are less than a handful of papers in the literature that have focused on combining SO and multi-objective problems. Most works either are single-objective or use a weighted sum method where objectives are transformed into a scalar single-objective by weighing them. Weighted sum approaches are ineffective in addressing complex problems due to their inefficient running time and limitations in finding specific Pareto-optimal solutions in the case of non-convex objective spaces.[38] In contrast, multi-objective methods do not require an a priori weighting of objectives.[39] They can find Pareto fronts or domains in the objective space of all the trade-off solutions rather than a single solution.[40] This paper applies a multi-objective approach to solve a stochastic version of ASP using SO. The key benefit of this approach is that all objectives are accomplished without having to make a compromise. In other words, all objectives are handled independently without negatively impacting each other. In addition, the proposed SO framework can incorporate the system's complexity and uncertainty.

From the technical point of view, the proposed SO model takes advantage of a newly developed simulation structure called *data-table input experiments*. Unlike the traditional simulation modeling that uses numeric *control variables* (i.e. number of beds, number of nurses, number of rooms, etc.), the data-table input models are designed to include data-tables as entries. Examples of data-tables are patients' arrival, nurses' work schedules, or even the sequence of patients' care delivery. MO-PASS considers the data-table as a control variable and tries to change the simulated system performance by optimizing the entire table entries.

Simulation modeling often uses a large amount of data to describe model entities, operations, resources, etc. In a healthcare setting, data-tables can include essential data such as patients' information (patient type, arrival time, patient's processing time in each unit, its corresponding physician, severity level), staff information (physicians/nurses work schedule), resources (personal protective equipment (PPE) supplies, order delivery lead time, minimum order quantity, order-level point), among many. To optimize table entries, one needs to define a parameter (control) for each cell entry of a table. This is not a trivial task and often requires a significant amount of time for setting and implementation. For instance, to optimize appointment scheduling in a healthcare system with 100 patients, the traditional SO approaches require 100 disjoint parameters to be manually defined—one parameter referring to each patient appointment. However, the Date-Driven Simulation-Optimization (DSO) approach simplifies this process by taking the whole table as a single parameter in the simulation model and optimizing the simulation output. This method has the following advantages:

- Easy implementation without the need to define explicit parameters;
- Tune a large set of parameters (which in this case are patients' appointments) simultaneously with no limitations;
- Deal with the scheduling of multiple patient types (or physicians) by defining a separate data-table input for each patient type (or physician) and determining the optimal scheduling pattern.

3. The proposed MO-PASS framework structure

To obtain a practical and ideal appointment scheduling solution, an integrated SO framework is developed using a simheuristic approach. Simheuristics are simulation-based optimization techniques in which a metaheuristic algorithm is applied as an optimization component to a simulation environment.[41] Its efficiency in solving combinatorial optimization problems with a stochastic nature made simheuristic a proper choice to tackle ASP in this paper. The proposed MO-PASS framework in this work integrates three modules: (1) optimization, (2) data exchange, and (3) simulation, as depicted in Figure 2.

In this sequential scheme, the multi-objective optimization module generates a solution (i.e. patients' appointment schedule). Then, the provided solution is transferred to the simulation module using a data exchange platform. At this point, the simulation model uses the provided solution in a simulated environment to evaluate the solutions, using their expected value for various metrics (i.e. healthcare system performance). These results are treated as the objective function values of the optimization problem. In other words, the simulation module is utilized as a *fitness function* of the metaheuristic algorithm. The optimization module leverages this feedback iteratively to evolve the solution and ultimately achieve an optimal or near-to-optimal patients' appointment schedule. The design aspects of the proposed MO-PASS framework are discussed as follows.

Table 1. A summary of related studies in the literature.

Studies	Objectives/Performance measures						Solution approach/Algorithm	Single/Multi-objective		
	Waiting time	Time in system	Resource utilization	Completion time	Throughput	Overtime		Single	Weighted sum	Multi-objective
Romero et al.[20]		✓					OptQuest; Integer linear programming and solved using a heuristic method	✓		
Dehghanimohammadabadi et al.[28]		✓					Simulation–optimization with Simulated Annealing (SA)	✓		
Azadeh et al.[14]		✓					Mixed-integer programming model solved using Genetic Algorithm	✓		
Pérez et al.[21]	✓		✓		✓		Stochastic Integer programming and Monte Carlo simulation	✓		
Leeftink et al.[16]	✓		✓			✓	Stochastic integer program and solved with a sample average approximation approach		✓	
Rezaeiahari and Khasawneh[15]	✓						Hybrid particle swarm optimization algorithm equipped with local search and a heuristic mining algorithm		✓	
Rezaeiahari and Khasawneh[32]		✓					Combination of simulation and TS		✓	
Klasses and Yoogalingam[26]	✓		✓			✓	OptQuest; Combination of scatter search and TS heuristics to search for feasible solutions and includes a neural network		✓	
Chien et al.[12]	✓		✓				An evolutionary approach based on a genetic algorithm		✓	
Saremi et al.[30]	✓			✓			Simulation-based Tabu Search (TS), Integer programming enhanced TS, and binary programming enhanced TS		✓	
White et al.[19]	✓		✓	✓			Full-factorial ANOVA	–	–	–
Harper and Gamlin[18]	✓	✓	✓				Simulation modeling	–	–	–
Saremi et al.[30]	✓			✓			Mathematical programming, simulation, and multi-objective TS methods			✓
Baesler and Sepuä lveda[31]	✓		✓	✓			Integration of simulation, goal programming, and genetic algorithm			✓

(continued)

Table 1. (continued)

Studies	Objectives/Performance measures						Solution approach/Algorithm	Single/Multi-objective		
	Waiting time	Time in system	Resource utilization	Completion time	Throughput	Overtime		Single	Weighted sum	Multi-objective
Jericä and Figueira[13]	✓		✓		✓		Variable neighborhood, scatter search-based method, and non-dominated sorting genetic algorithm (NSGA-II)			✓
This paper		✓		✓		✓	Data-driven multi-objective simulation–optimization model using MOPSO			✓

ANOVA: analysis of variance; MOPSO: Multi-Objective Particle Swarm Optimization.

3.1. Optimization module

A multi-objective optimization problem with $m(\geqslant 2)$ conflicting objectives $f_i : \Re^n \rightarrow \Re$ is defined as follows:

$$\textit{Minimize } \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})\}$$

subject to $\mathbf{x} \in \mathcal{S}$. The *decision vector* $\mathbf{x}$ belongs to the feasible region $\mathcal{S} \subset \Re^n$. A feasible decision vector $\mathbf{x}_1 \in \mathcal{S}$ is said to (Pareto) *dominate* another decision vector, $\mathbf{x}_2 \in \mathcal{S}$ (denoted $\mathbf{x}_1 \prec \mathbf{x}_2$) if:

1- The decision vector $\mathbf{x}_1$ is no worse than $\mathbf{x}_2$ in all objectives:

$$f_i(\mathbf{x}_1) \leqslant f_i(\mathbf{x}_2), \quad \forall\, i \in \{1,\ 2,\ \ldots, m\}$$

2- The decision vector $\mathbf{x}_1$ is strictly better than $\mathbf{x}_2$ in at least one objective:

$$f_j(\mathbf{x}_1) < f_j(\mathbf{x}_2), \quad \exists\, j \in \{1,\ 2,\ \ldots, m\}$$

The graphical representation of these definitions is represented in Figure 3, where $m = 2$. This illustration shows an example of a Pareto frontier (in red) and the set of Pareto-optimal solutions (non-dominated solutions) in boxed points. Point j is not on the Pareto frontier because it is dominated by point i. Points $i - 1$, i, and $i + 1$ are not strictly dominated by any other, and therefore are included on the frontier. The crowding distance shows how close an individual is to its neighbors.[42] To maintain solution diversity, the larger crowding distance is better[43] because it reveals more valuable information about its area.

The applied optimization algorithm in this framework is *multi-objective particle swarm optimization*, referred to as MOPSO. This algorithm is an extension of *particle swarm optimization* (PSO), a metaheuristic algorithm for multi-objective optimization problems. MOPSO is a population-based algorithm that uses an external memory (called repository) and a geographically based approach (called crowding distance) to maintain the diversity of solutions.[44] This algorithm has shown competitive performance in multi-objective optimization problems[45] and has been applied to solve many scheduling[46–48] and healthcare-related problems.[49–51]

Unlike conventional optimization techniques (i.e. single-objective optimization), multi-objective metaheuristics (particularly MOPSO) can efficiently improve all objectives simultaneously without negatively impacting each other. In addition, in these methods, there is no need to define coefficients (weights) to aggregate objectives because all objectives are treated independently. As a result, MOPSO was selected as the engine of the multi-objective optimization module to be coupled with the

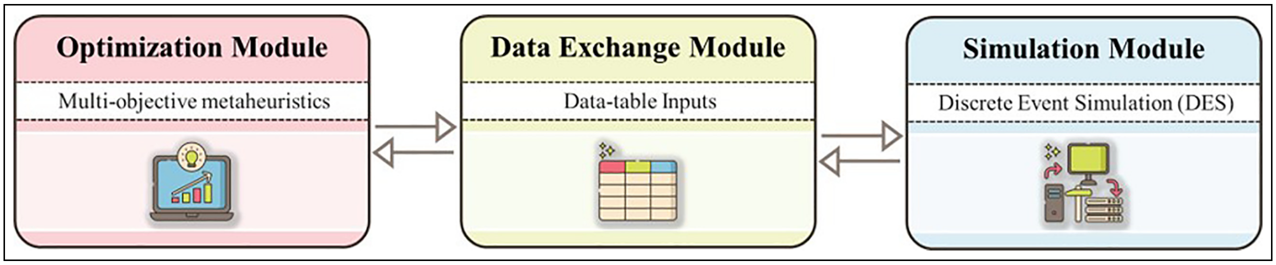


Figure 2. The proposed MO-PASS framework structure.

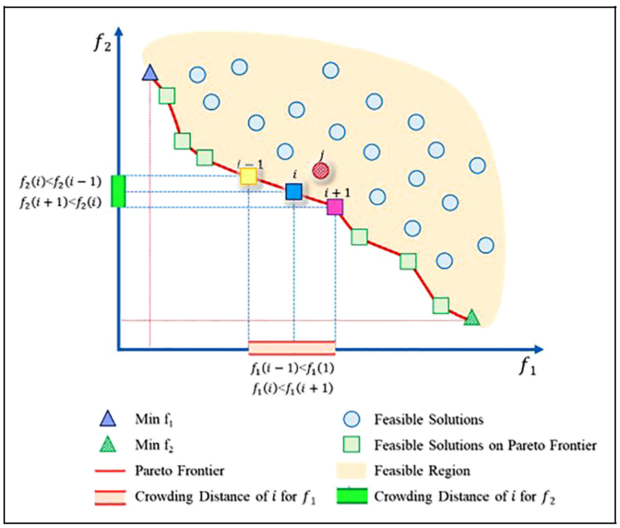


Figure 3. An example of a Pareto frontier (in red) and the set of Pareto-optimal solutions (non-dominated solutions).

simulation model. We refer interested readers to Coello and Lechuga[44] for a detailed exposition of the MOPSO algorithm. For a detailed problem formulation, please see Appendix 2.

3.2. Simulation module: DES

DES is the dominant simulation approach for healthcare problems[52] and has been successfully used in a variety of healthcare settings. Using DES, practitioners can efficiently analyze the system behavior[53] and find operational solutions before implementation.[54]

Under the taxonomy of operational problems in healthcare, simulation has been tremendously used for patient scheduling to improve resource efficiency, quality, and patient access. As a powerful tool, DES can deal with well-known uncertain factors in appointment scheduling. These factors are environmental factors (multiple patient types, multi-stage service processes, multiple providers, etc.) and patient and provider-related behavioral factors (patient no-show and cancelation, selection behavior for time slots and doctors, timeliness of doctors, inconsistent service rate of doctors, etc.).[55] Also, the existing software packages have been increasingly adapted to healthcare through enhanced visualizations and modeling.[56] Therefore, DES is applied in this study to imitate the system details and operations accurately.

3.3. Data exchange module

In the applied DSO structure, controls are data-table entries rather than numerical variables or model parameters. This allows multiple model entries to be tested and optimized simultaneously, which in this case are appointment times. In this algorithm, the simulation model starts with an arbitrary solution (random appointment times), which is then improved iteratively by the optimization module to obtain the optimal solution (the best patient appointment schedule).

4. Framework implementation

Three software packages are integrated to implement the proposed MO-PASS model. As illustrated in Figure 4, the optimization module in MATLAB is connected with the simulation module in Simio through an MS Excel file as a data exchange platform.

MATLAB has many advanced capabilities and can be easily linked to a variety of different software packages, including data exchange platforms and simulation tools. To model the system's operations, Simio, a simulation software is utilized. Simio is flexible enough to model healthcare systems and their operational details. Its API capability enables researchers to program customized logic or extend Simio's access to external software packages. In previous works,[53,58,59] the authors took advantage of this feature to connect MATLAB and Simio to develop different types of SO models. As a result, the optimization module, MOPSO, was manually coded in MATLAB and then linked to the Simio to establish the SO framework.

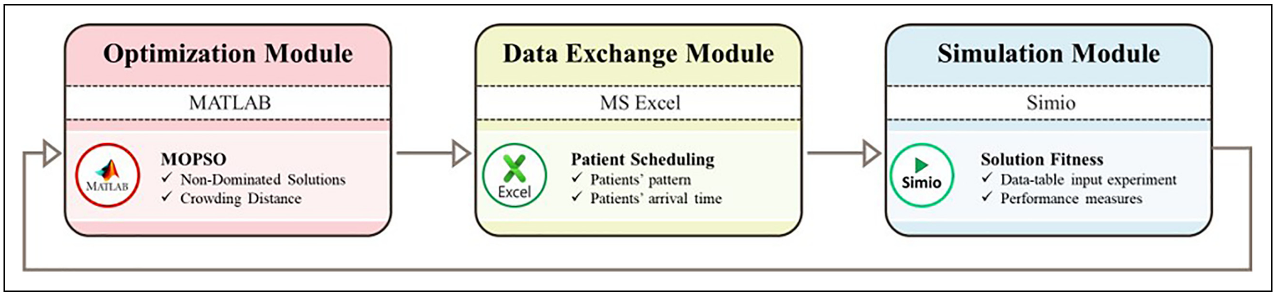


Figure 4. Structure of the proposed MO-PASS simulation–optimization framework.

Excel is the interface between the optimization and simulation modules. MATLAB generates a new solution instance in each iteration of the framework and directly writes it into Excel. This solution includes two columns: (1) *patients' arrival time* and (2) *patient types*. The fitness of this solution is then tested in a simulation environment.

Therefore, Simio considers the Excel data as a *data-table input* and updates the model configuration (patients' appointments) upon its initialization. As depicted in Figure 5, Simio creates patient entities (arrival times) and determines their type from the first and second columns of the data-table input correspondingly. Using this approach, Simio can instantly simulate the model and determine how the new appointment schedule impacts the model objective functions. Then, the estimated objective function values derived from the simulation module are transferred to the optimization module to start a new iteration and generate a new instance of the solution. All the framework components, such as MOPSO Algorithm and API connection between Simio and MATLAB, are coded manually. Details of these modules and their interactions are provided as follows.

5. Case study: an outpatient cancer clinic appointment scheduling

According to the World Health Organization (WHO), breast cancer is one of the most diagnosed cancers worldwide and is the second leading cause of death among women. Among every eight women in the United States, one is diagnosed with breast cancer in her lifetime. Deaths due to breast cancer have slowed since the 1990s due to better screenings, early detections, increased awareness, and improved treatment options.[60] Breast cancer diagnostic services are more streamlined than ever, and the patient can have frequent visits with multidisciplinary teams.[61] As part of the standard diagnostic breast cancer process, known as "triple assessment," patients receive clinical breast examinations, breast imaging (a combination of mammography and ultrasound, or both), and needle biopsies when necessary.[62] Studies show that even patients

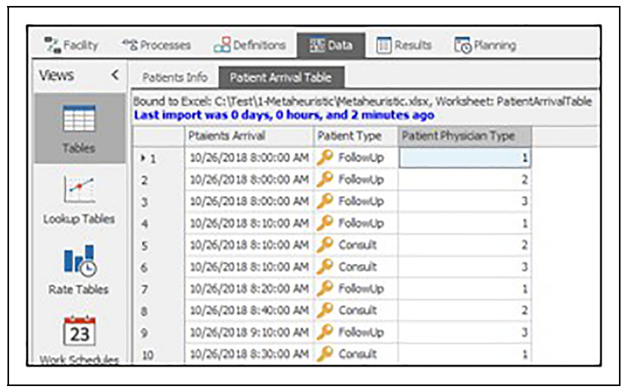


Figure 5. Data-table input of patients' appointment in Simio environment.

who were assessed normal or begin diagnosis might develop internal cancer before their next screening invitation.[62] Therefore, efficient delivery of diagnostic breast cancer services is essential, especially for patients who need routine imaging or frequent visits with the surgeon.

To evaluate the effectiveness of the proposed model, the MO-PASS framework is applied in a diagnostic breast cancer clinic with three patient types: the first patient type is (1) surgical *follow-up patients* who require routine imaging and frequent visits with the surgeon, (2) *consult patients* who come for lump evaluations and guided biopsy needs, and (3) diagnosed patients who seek second opinion who might not need a biopsy, called *second-opinion patients*. The purpose of second-opinion patients' visits is to review images and confirmation of diagnosis.

To solve this problem, the optimization module considers two factors to generate appointment schedules. First, (1) it determines the patients' arrival pattern to the model, and second, (2) it calculates the inter-arrival time between patients. The arrival pattern indicates the sequence of patient types that arrive at the system, and the inter-arrival time shows the time interval between different patients. The combination of these two results will provide an appointment scheduling of patients. To clarify the model procedure, an example of a given solution is represented in Appendix 3.

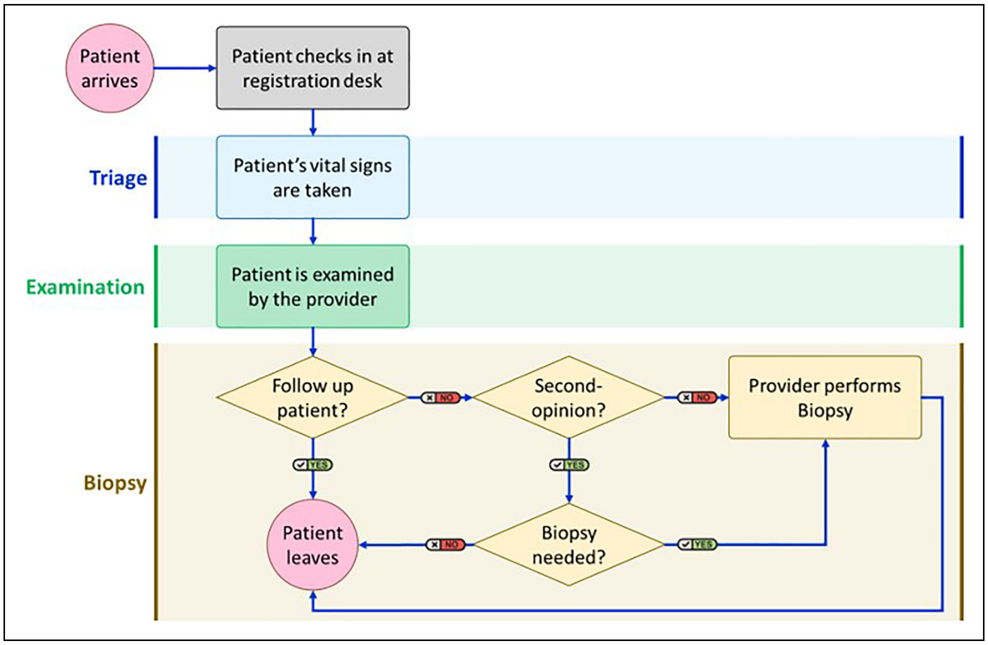


Figure 6. Patient flow at an outpatient cancer clinic with three patient types (follow-up, consult, and second opinion).

Table 2. Patient mix percentages and biopsy probability.

Patient type	Percentage	Biopsy probability (%)
Follow-up	55	0
Second opinion	40	70
Consult	5	100

5.1. Simulation model characteristics

To model the patient flows and operations, a DES model is created in Simio. Figure 6 illustrates the sequence of each patient type in different stages. In this process, all patients are first triaged by a single Medical Assistant (MA). Then depending on the patient types, different patients will receive different types of care. A follow-up patient is examined by a physician and leaves the clinic. A consult patient is examined by a physician and then is taken to the biopsy room to perform the biopsy. When the consult patient leaves the biopsy room, the provider returns to the examination room to see the next patient. After being triaged by an MA, the second opinion patient is also examined by a physician. Not all second-opinion patients need a biopsy; therefore, depending on the physician's opinion, these patients may require a biopsy which will be taken with the same physician. Several assumptions are made to model this problem:

1. There will be a smaller number of second-opinion patients compared to follow-up and consult patients. The distribution of the number of patients is according to Table 2.
2. The probability threshold for the physicians to decide whether to perform a biopsy for the second-opinion patients is 0.7.
3. Physicians are equally assigned to the number of patients.
4. The clinic is open from 8:00 am to 5:00 pm.
5. There are three physicians, and each has its examination room, and they all share a separate room to perform the biopsy.
6. A medical office assistant triages all patients.
7. Service times are stochastic and are determined based on expert opinion (Table 3). The distributions used in the model are based on the provided data by the clinic. In case of insufficient data, experts' opinion (after explaining how each distribution would manifest in the inter-arrival and service times) is used. So, the applied distributions are more relevant to the provided data and intuitive for the clinic experts. Interested readers can refer to the works by Shaaban and Hudson[63] and Romero-Silva et al.[64] for discussions on distribution fitting, and Jáuregui et al.[65] and Altiok and Melamed[66] for parameter selection of a healthcare system modeling.

The objective functions of the model are calculated based on the simulation results. The Total Service Time, or TST, is the time a patient spends in the healthcare

Table 3. Patients' service time distributions.

Procedure	Duration (min)		
	Follow-up	Second opinion	Consult
Triage	Triangular (10, 12, 17)	Triangular (10, 12, 17)	Triangular (10, 12, 17)
Examination	Triangular (14, 17, 20)	Triangular (15, 22, 25)	Triangular (14, 20, 27)
Biopsy	Uniform (10, 15)	Uniform (10, 15)	Uniform (10, 15)

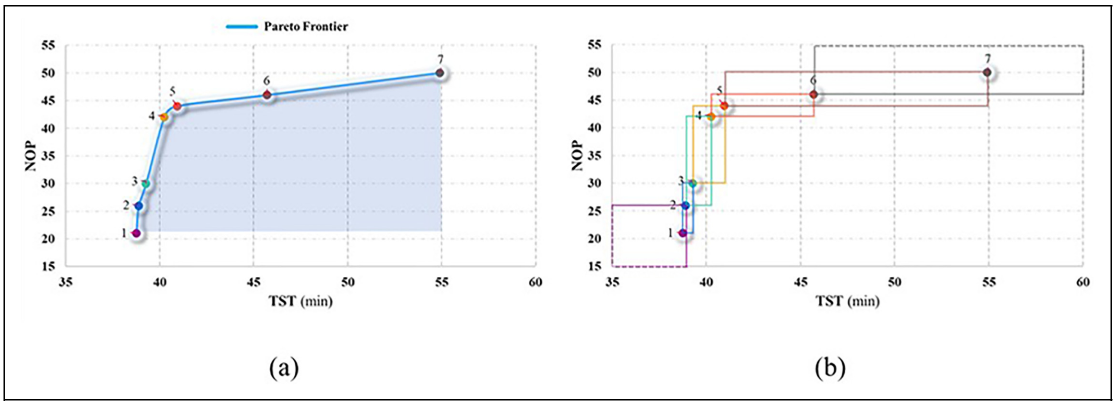


Figure 7. Illustration of (a) non-dominated sorting Pareto frontier and (b) crowding distance of solutions obtained by MO-PASS using MOPSO algorithm.

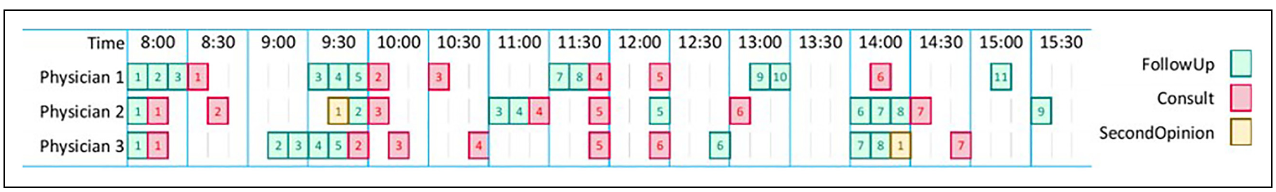


Figure 8. A sample appointment scheduling "template" generated by MO-PASS framework (solution 7).

center. This measure is calculated based on the elapsed time between the patient's arrival and departure time. The second objective function is the number of visited patients, or NOP. This refers to the total number of patients who could receive their care by the end of the clinic's operating hours. All patients must leave the system by the end of the day.

5.2. *Experimental analysis*

This section presents the empirical results of the proposed MO-PASS model. First, the analytical results of the optimization model and its Pareto front solution are discussed. Then, to evaluate the effectiveness of the MO-PASS model, a computational experiment is designed to compare its performance against a heuristic approach.

5.2.1. MO-PASS experiment results. The proposed MO-PASS model was applied to solve the multi-objective case study with two conflictive objective functions. The model aims to minimize TST and maximize NOP with zero overtime. As shown in Figure 7(a), the simheuristic model could solve the problem and provide six (6) non-dominated solutions as a part of the Pareto frontier. These solutions are a trade-off between objectives and are not dominated by any other solutions. In one extreme, the best-obtained solution based on TST was 38.75 min (solution 1 with NOP = 21); on the other extreme, the maximum NOP was obtained by solution 7 with NOP = 50. Figure 7(b) shows the crowding distance of each of the solutions in the frontier. As mentioned earlier, the larger the crowding distance, the better. Solutions 1, 6, and 7 have the largest crowding distance.

Also, Figure 8 demonstrates a sample solution by the model. Remember, the model aims to provide a template of appointments, determining the allocated time slot for different patient types (and not the individual patients). The scheduling staff can use the provided template to

Table 4. Patient sequencing based on three different heuristic rules: SPT, LPT, and DSR.

Patients	Service time (h)	Ordered service time			Patient-type order		
		SPT	LPT	DSR	SPT	LPT	DSR
P1	5	1	7	2	P5	P3	P2
P2	2	2	5	5	P2	P1	P1
P3	7	3	3	7	P4	P4	P3
P4	3	5	2	3	P1	P2	P4
P5	1	7	1	1	P3	P5	P5

SPT: shortest processing (service) time; LPT: longest processing time; DSR: dome-shape rule.

schedule appointments for patients upon their call or request. The patient-type mix in this solution was predetermined (55% follow-up, 40% consult, and 5% second opinion), and therefore, each patient type received a different number of allocated time slots proportionally. In this specific example, it is assumed physicians' load is equal (all three physicians have almost the same number of patients) while they share resources and equipment.

In addition, these results are an indicator of a successful implementation of MO-PASS. The model efficiently generates appointment templates for multiple patients and physicians using multi-objectives. The Pareto frontier allows a posteriori selection of a solution and balance between objectives with further insights. More importantly, MO-PASS enabled conducting the experimentation with minimal efforts and adjustments on the simulation model. Rather than defining an enormous number of decision variables (controls), MO-PASS applied changes to the data-table inputs and optimized patient appointment templates.

5.3. Comparing MO-PASS with heuristic approaches

To evaluate the performance of the MO-PASS model, its results are compared with some heuristic appointment scheduling approaches found in the literature. These approaches are formed based on two elements: (1) adjusted service time and (2) scheduling rule.

The adjusted service time indicates the arrival time of the patient to the system or, basically, its appointment time. The scheduling rule specifies how patients are prioritized to receive care. One of the well-known adjusted service time methods is the *job hedging* approach which is applied in many appointment scheduling studies, including the works by Saremi et al. and Gul et al.[29,67] In this method, the adjusted service time S' is calculated based on Equation (1):

$$S' = \mu + \alpha\sigma \tag{1}$$

where μ is the mean of service time, σ is the standard deviation of service time, and α is a real number between -1 and 1 ($\alpha\epsilon[-1, 1]$) that determines the tightness of service time variability.

Three different sequencing rules that are proposed in the literature are combined with various hedging levels to complete heuristic approaches. These rules are (1) SPT, or shortest processing (service) time; (2) LPT, or longest processing time; and (3) dome-shape rule (DSR) originally proposed in the work by Robinson and Chen.[7] In DSR, the patient with the largest service time is placed in the middle of the sequence, and patients with lower service time are placed before and after it alternatively. The rationale of DSR is that patients in the middle of the day are more likely to experience waiting time and warrant additional planned service time since they have the potential to disrupt the remainder of the schedule when waiting occurs.[68]

To better explain these rules, an example of sequencing is provided, including five different patient types with different average service times [$\mu_{P_1} = 5$, $\mu_{P_2} = 2$, $\mu_{P_3} = 7$, $\mu_{P_4} = 3$, $\mu_{P_5} = 1$]. As shown in Table 4, patients are sorted according to the mean service time by ascending (SPT) and descending (LPT), while in DSR, patients with longer service times are placed in the middle of the sequence, and ones with shorter service time at the beginning or end of the sequence. A graphical representation of these sequencing methods is provided in Figure 9.

5.3.1. Results analysis. The obtained results from the simheuristic MO-PASS model using MOPSO are compared with the heuristic as mentioned in the earlier approaches. To have an even ground for comparisons, similar simulation models with identical distributions were used in all scenarios. Table 5 summarizes the solution approach algorithms and settings. Multiple experiments are conducted to find a desirable performance for the metaheuristic algorithm. The results of this experiment are analyzed with respect to the objective functions of the model.

To conduct the comparison, the best results of Job Hedging are compared with MO-PASS Pareto frontier solutions. Figure 10 illustrates an instance of results where the solution with the best TST obtained from Job Hedging (h_best_TST) is dominated by three MO-PASS results (points 3, 4, and 5). As depicted in Figure 11, the superiority of the MO-PASS is evident from the simulation results (replication size = 100). Figure 11(a) confirms the three

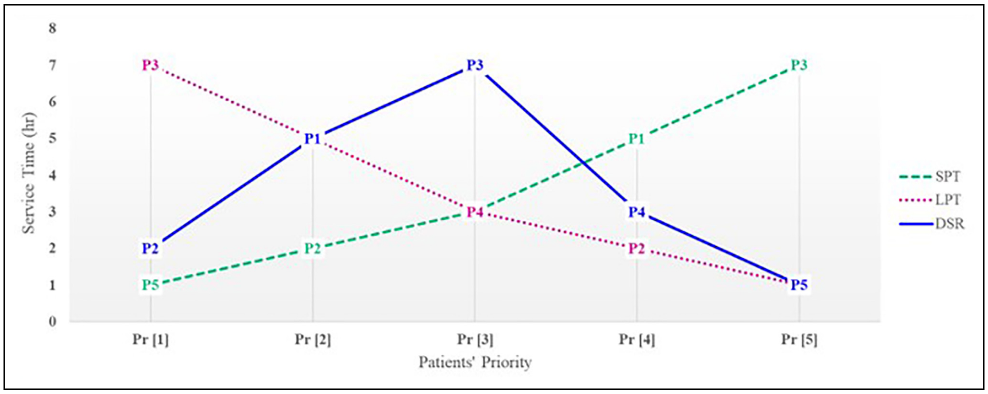


Figure 9. A numerical example of patients sequencing based on SPT, LPT, and DSR heuristics.

Table 5. Experiment setting for the simheuristic and heuristic models.

Solution approach	Algorithm	Settings	
Simheuristic model (MO-PASS)	MOPSO	Max iteration	20
		Population size	30
		Repository size	15
		Inertia weight	0.5
		Personal learning coefficient	1
		Global learning coefficient	2
		Number of grids per dimension	5
		Leader selection pressure	2
		Deletion selection pressure	2
		Mutation rate	0.1
Heuristic models	Job Hedging	$\alpha\epsilon[-0.8,0.8]$	
	Sequencing rules	SPT, LPT, DSR	

MO-PASS: Multi-Objective Patient Appointment Scheduling; MOPSO: Multi-Objective Particle Swarm Optimization; SPT: shortest processing (service) time; LPT: longest processing time; DSR: dome-shape rule.

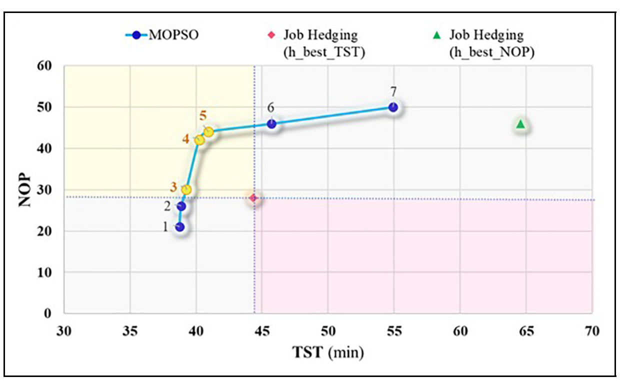


Figure 10. Comparing MO-PASS Pareto frontier with Job hedging results (best TST—red diamond point).

MO-PASS solutions or repository members (rep 3, rep 4, rep 5) not only have a better TST performance but also have less variability and are more reproducible. This figure depicts the boxplot of the solutions in addition to each replication observation (100 points). Figure 11(b) emphasizes the number of scheduled patients in a day, where all of MO-PASS achieved a better result (NOP $\geqslant$ 30) compared with the best heuristic solution, h_best_TST, with NOP = 28.

Figure 12 provides the comparative results from the NOP point of view. This figure shows how two MO-PASS frontier solutions (points 6 and 7) dominate the best number of patient solutions obtained by the heuristic approach (h_best_NOP). Figure 13 depicts more details where schedules provided by solutions 6 and 7 perform significantly better than the heuristic model's best NOP solution. Part (a) of this figure highlights TST simulation results, where part (b) presents NOP outcomes.

In summary, computational results indicate that the proposed MO-PASS model outperforms existing heuristic approaches for appointment scheduling. The main advantage of MO-PASS is using a *multi-objective scheme* and an *evolutionary structure*. Using MOPSO as the multi-objective optimization scheme allows the model to improve the objective functions simultaneously without degradation of the other objectives. In this algorithm,

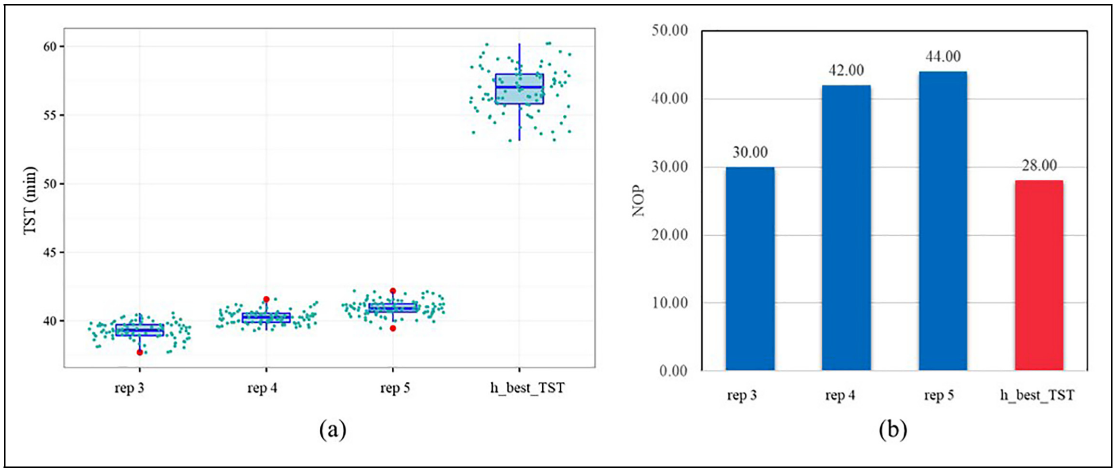


Figure 11. Non-dominated MO-PASS results compared with the best TST heuristic solution: (a) boxplot of simulated results for TST with 100 replications, (b) bar chart of the number of scheduled patients (NOP). Please note, rep 3, rep 4, and rep 5 are three solutions provided by MO-PASS shown in Figure 10.

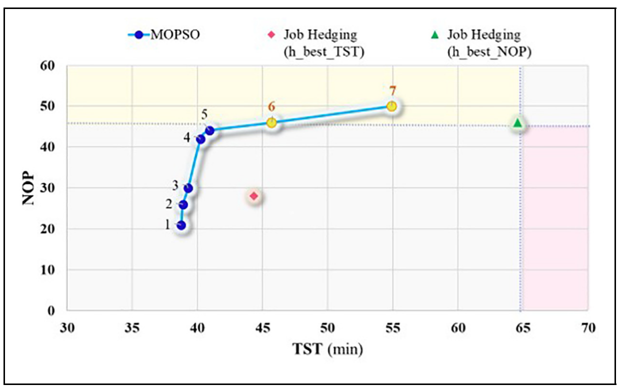


Figure 12. Comparing MO-PASS Pareto frontier with Job Hedging results (best NOP).

tuning objective coefficients (weights) is no longer a concern; therefore, the provided results are a worthwhile trade-off of the objectives. Also, the evolutionary structure of the model enables a progressive and selective discovery of non-dominated solutions. This allows constant improvement of the solutions in situations where the algorithm can run for longer times.

6. Conclusion and future works

Appointment scheduling systems are developed to help healthcare facilities improve the quality of care and overall patient satisfaction. Many interplaying factors are involved when designing an efficient PASS. Due to the stochastic nature of healthcare systems, SO is a well-suited approach to deal with ASPs incorporating sources of uncertainty. This study tries to understand these critical factors and propose a practical decision-making system to assign appointments to patients, which is called MO-PASS.

A data-table experiment technique is used to solve the problem. This SO model combines a multi-objective meta-heuristic algorithm, namely MOPSO, and a DES environment. Three software packages, MATLAB, Simio, and MS Excel, were linked together to deploy this framework. From the practical point of view, implementing MO-PASS was straightforward, and applying the data-table input technique made the proposed MO-PASS framework easy to develop.

To show the applicability and capability of MO-PASS, an appointment scheduling of a cancer clinic was considered as the case study. The goal of the model was to find a balance between the system's throughput, patients' time in the system, with no overtime. The optimal solution to this problem was an efficient schedule that considered these factors to improve the level of service by considering patents flow and physician availability. In an experimental setting, the performance of MO-PASS was compared with common heuristic appointment scheduling approaches found in the literature. Computational results showed the superiority of MO-PASS compared to these heuristics for all objective functions.

The main novelty of MO-PASS was in its practicality and bridging the gap between existing models and algorithms in PAS. Future works can leverage this framework and extend it in many different directions. One possibility is to drop the assumptions in this study and include them in the problem formulation. For instance, considering no-show patients or walk-ins can make the problem more realistic and practical. Also, it was assumed that all physicians could treat all types of patients, while in some cases, this may not be viable. Therefore, specific assignment rules may need to be considered to accurately assign patients to physicians.

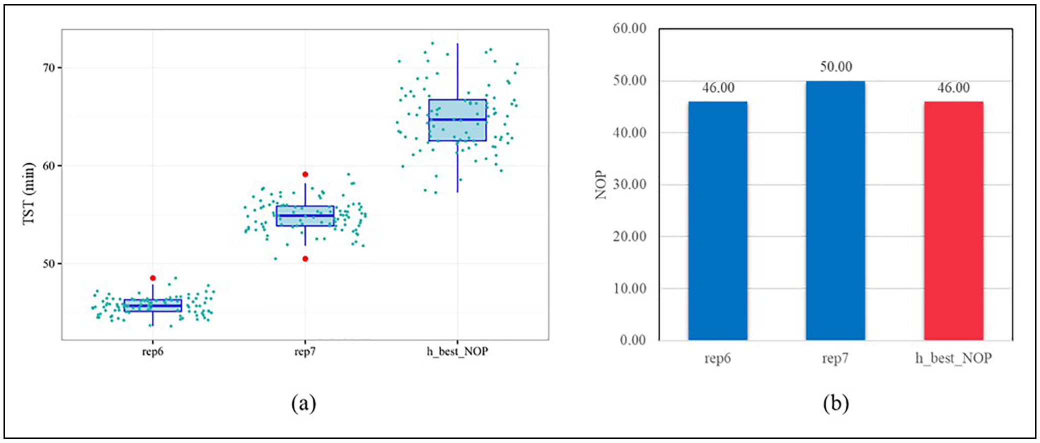


Figure 13. Non-dominated MO-PASS results compared with the best NOP heuristic solution: (a) boxplot of simulated results for TST with 100 replications, (b) bar chart of the number of scheduled patients (NOPs). Please note, rep 6, and rep 7 are solutions provided by MO-PASS shown in Figure 12.

Moreover, the presented study applied the MO-PASS framework to solve a multi-stage ASP. However, one can use the same framework to address single and multi-ASPs. This can be easily implemented due to the data-table property of the model. And finally, future research works can include state-of-the-art multi-objective evolutionary algorithms, such as NSGA-II,[69] SPEA2,[70] or IBEA,[71] to compare their performance on forming Pareto front solutions.

Appendix 1

List of abbreviations

Table 6. List of abbreviations.

Abbreviation	Description
AS	Appointment Scheduling
ASP	Appointment Scheduling Problem
DES	Discrete Event Simulation
DSR	Dome-Shape Rule
IP/MIP	Integer Programming/Mixed Integer Programming
LPT	Longest Processing Time
MA	Medical Assistant
MO-PASS	Multi-Objective Patient Appointment Scheduling System
MOPSO	Multi-Objective Particle Swarm Optimization
NOP	Number of Patients
PASS	Patient Appointment Scheduling System
PSO	Particle Swarm Optimization
SBO	Simulation-Based Optimization
SO	Simulation–Optimization
SPT	Shortest Processing Time
SP	Stochastic Programming
TST	Total Time in System

Appendix 2

Problem formulation

In this appendix, the problem is formulated as a two-stage stochastic programming model to have a precise definition of the assumptions, constraints, and objective functions. Note that there is a difference between the presented problem and conventional scheduling problems. Here, the focus is on job types, in this case, *patient types*, and not individual jobs (patients). The following notations are used throughout the mathematical formulation. The two-stage stochastic program is presented next.

Sets and indices

P	Number of all patients requesting a visit on a given day
C	Set of all patient types
O	Set of all types of processes the patients may go through
Ω	Set of all scenarios
p	Index for the individual patients, $p = 1, \ldots, P$
n	Index for positions in a sequence of the patients, $n = 1, \ldots, P$
c	Index for patient types, $c \in C$
o	Index for processes, $o \in O$
ω	Index for scenarios, $\omega \in \Omega$

Parameters

D_{po}^{ω}	Duration of service to patient p regarding process o under scenario ω
G_{pc}	Binary parameter which takes the value 1 if patient p is of type c, and 0 otherwise. $\sum_{c \in C} G_{pc} = 1, \ \forall p = 1, \ldots, P$ (each patient can be of one and only one type)

T	Length of the time period over which the appointments are scheduled
Q_c	Minimum percentage of the total number of patients of type c to be served (requirement)
K	A sufficiently large number

First-stage decision variables

$x_{ci} \in \mathbb{Z}^+$	Inter-arrival time for patient type c scheduled after patient type $i \in C$
$z_n \in \mathbb{Z}^+$	The scheduled start time of the nth patient
$l_{pn} \in \{0, 1\}$	Binary variable that takes the value 1, if patient p is scheduled for appointment slot n (as the nth patient), and 0, otherwise
$y_p \in \{0, 1\}$	Binary variable that takes the value 1, if patient p will be seen, and 0 otherwise

Second-stage decision variables

$s^{\omega}_{pn} \in \mathbb{R}^+$	Actual start time for patient p under scenario ω, if he or she is scheduled as the nth patient
$w^{\omega}_{pn} \in \mathbb{R}^+$	Waiting time of patient p under scenario ω, if he or she is scheduled as the nth patient
$TST \in \mathbb{R}^+$	Total expected patients' total time in the system (the sum of all waiting and processing times)

Two-stage stochastic program

The objectives of this problem are to minimize the total time in the system (TST) and to maximize the total number of patients (NOP) seen. TST comprises two main components: the waiting time before check-in and the total time spent for various processes. We minimize the *expected* value of the TST because the waiting time depends on how long it takes for the previous patients to complete their processes, which is probabilistic. The main assumption here is that in each facility/room of the hospital/clinic, patients are seen one at a time. In other words, each room/doctor's capacity is only one patient; a new patient is seen once the last patient checks out. This is similar to the fallow shop scheduling problem, where a machine processes only one job and becomes available for the next job once the current job goes to the next machine. Here, the jobs are patients, and the machines are rooms, doctors, or, generally speaking, various stages within healthcare facilities. The objective function is defined below, subject to the following constraints. Some of these constraints are non-linear. Although their linearization does not require significant effort, we present them in the current non-linear form for simplicity, as the main purpose of this mathematical formulation is to precisely define and present the problem. Another assumption is that our model does not consider walk-in patients, which may or may not be the common practice:

$$OF_1: \ minimize\ LOS = E_{\omega\in\Omega}\left[\sum_{p=1}^{P}\left(\sum_{n=1}^{P} w^{\omega}_{pn} + \sum_{o\in O} D^{\omega}_{po}\right)\right] \tag{2}$$

$$minimize\ LOS = \sum_{\omega\in\Omega} \Pr(\omega) \sum_{p=1}^{P}\left(\sum_{n=1}^{P} w^{\omega}_{pn} + \sum_{o\in O} D^{\omega}_{po}\right) \tag{3}$$

$$OF_2: \ maximize\ NOP = \sum_{p\in P} y_p \tag{4}$$

TST in Equation (1) is calculated by adding total processing times to the total waiting times for each patient. However, note that the second part (sum of all processing times) is a constant, which is removed in Equation (2). The NOP seen in the scheduling time in Equation (3) can be obtained by adding all the y_p's as they indicate if a patient has been seen before time T. Both objectives are subject to the following constraint.

- Minimum service requirement

$$\sum_{p=1}^{P} y_p G_{pc} \geqslant Q_c P \qquad \forall c \in C \tag{5}$$

- Scheduling time limit

$$\left(y_p - 1\right)K \leqslant T - \sum_{n=1}^{P} z_n l_{pn} \qquad \forall p = 1, \ldots, P \tag{6}$$

- Slot-to-patient allocation

$$\sum_{n=1}^{P} l_{pn} = 1 \qquad \forall p = 1, \ldots, P \tag{7}$$

- Patient-to-slot allocation

$$\sum_{p=1}^{P} l_{pn} = 1 \qquad \forall n = 1, \ldots, P \tag{8}$$

- Setting the start time of the first slot/patient to 0

$$z_1 = 0 \tag{9}$$

$$s^{\omega}_{1p} = 0 \qquad \forall p = 1, \ldots, P, \omega \in \Omega \tag{10}$$

- Slot start-time calculation

$$z_n = z_{(n-1)} + \sum_{p=1}^{P} l_{pn}\left(\sum_{i=1}^{P} l_{i(n-1)} \sum_{c=1}^{C}\sum_{j=1}^{C} x_{cj} G_{pc} G_{ij}\right), \quad \forall n = 2, \ldots, P \tag{11}$$

- Patient start-time calculation

$$s_{pn}^{\omega} = \sum_{i=1}^{P}\left(s_{i(n-1)}^{\omega} + \sum_{o \in O} l_{i(n-1)} D_{io}^{\omega}\right) \quad (12)$$
$$\forall p = 1, \ldots, P,\ \forall n = 2, \ldots, P\ , \omega \in \Omega$$

- Patient waiting time

$$w_{pn}^{\omega} \geqslant s_{pn}^{\omega} - \sum_{n=1}^{P} z_n l_{pn} \qquad \forall p, n = 1, \ldots, P,\ \omega \in \Omega \quad (13)$$

- No overtime

$$\sum_{p=1}^{P} l_{pP}\left(s_{pP}^{\omega} + \sum_{o \in O} D_{po}^{\omega}\right) \leqslant T \quad (14)$$

Constraint set (5) ensures that a preset minimum percentage of all patients are of a certain type. y_p determines whether patient p is seen, which is multiplied by G_{pc} to make sure they will be counted only if they are of type c. Constraint set (6) forces y_p to take the right value depending on whether the patient is scheduled to be seen before or after the finish time (T). Constraint sets (7) and (8) ensure that each patient is assigned to only one position in the sequence (queue) of all the patients, and only one patient is assigned to each position in the queue. Constraint (9) sets time 0 as the scheduled start time of the first patient (a first-stage constraint), while Constraint (10) does it for whoever is scheduled first in the second stage. Constraint sets (11) and (12) calculate the start time of the next patients in the queue based on the start time of the previous patients and how long they stay to complete their processes. Waiting times are calculated in Constraint set (13) as the difference between the scheduled start time and the actual start time for each scenario. If the scheduled time is less than the actual time, the optimization algorithm will assign 0 as the value of w_{pn}^{ω} because it is being minimized in the objective function. Constraint set (14) ensures that overtime is not allowed by restricting the finish time of the last patient to be less than or equal to the total time available (T).

Appendix 3

Solution representation of MOPSO

Any generated solution by the optimization model contains an array of numbers that will be parsed into an appointment scheduled following these steps:

1. *Determine patients' sequence.* For a given system with the capacity of P (max number of patients that can be visited per day) and C number of patient types, model generates an array with the size of P with integer values between $[1, C]$. Figure 14 demonstrates an example of this representation where $P = 8$ and there are three patient types, namely, follow-up, consult, and second opinion ($C = 3$). The solution initially includes a random number between 0 and 1, where then it is mapped to patient types $[1, C]$. In this example, patients are equally distributed, while in reality, patients' mix could have any distribution.
2. *Determine patient types' inter-arrival time.* Assuming there is a defined inter-arrival time between two patient types, i and j, with C number of patient types, there are C^2 number of possible inter-arrival times between patient types. Due to the operational aspects of the problem, inter-arrival times are selected from a list of numeric series between 10 and 60 min with 10-min intervals ($I = [10, 20, 30, 40, 50, 60]$). Figure 15 represents this step where $C = 3$, and therefore, there are $C^2 = 9$ different inter-arrival times between patient types. This example indicates the inter-arrival time between two follow-up patients ($i = 1$ and $j = 1$) is 50 min, while the inter-arrival time between a consult patient ($j = 2$) succeeded after a follow-up patient ($i = 1$) is 10 min.
3. *Generate the appointment scheduling.* Based on the provided patients' sequence and inter-arrival times from steps 1 and 2, the patient appointment schedule can be created. As shown in Figure 16, patients are sequenced based on the pattern determined in step 1. Then, the inter-arrival time of two consecutive patients is used to calculate a new

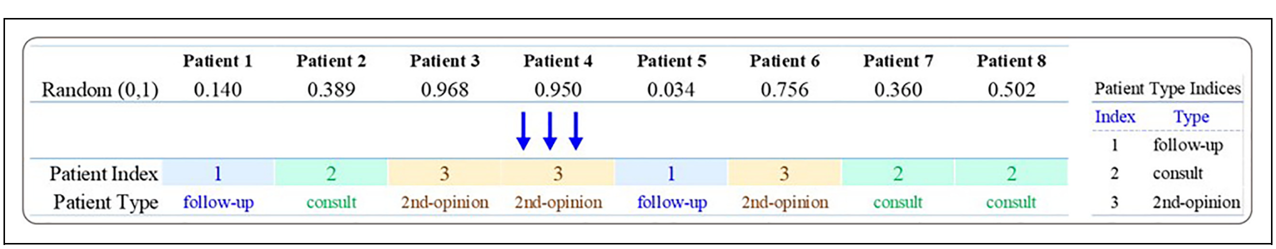
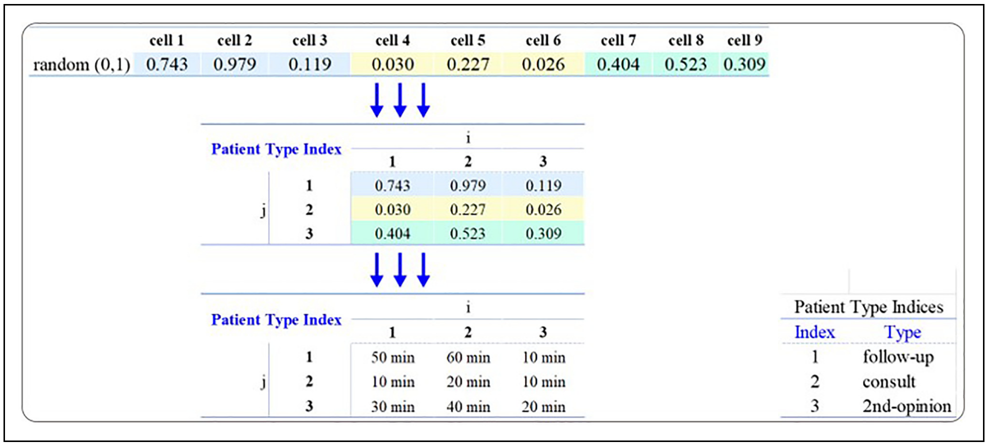


Figure 14. Solution representation of patient's sequence determination ($P = 8$, $C = 3$).



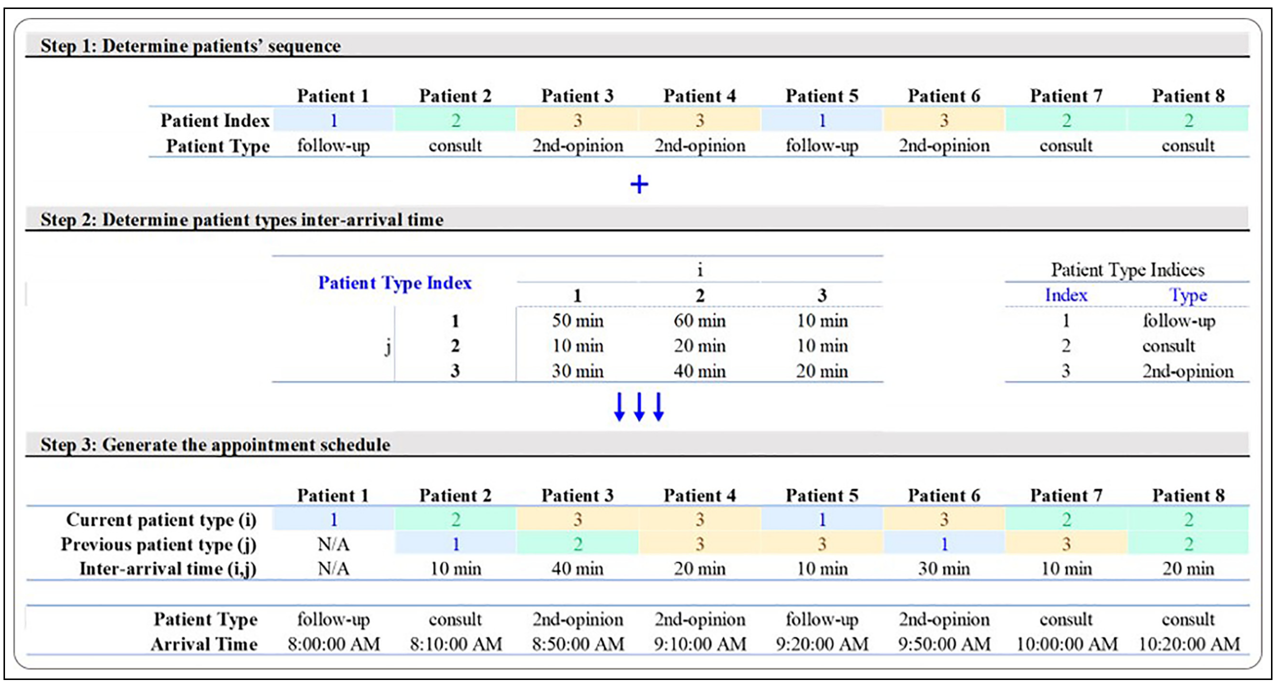
	cell 1	cell 2	cell 3	cell 4	cell 5	cell 6	cell 7	cell 8	cell 9
random (0,1)	0.743	0.979	0.119	0.030	0.227	0.026	0.404	0.523	0.309

Patient Type Index		i = 1	i = 2	i = 3
j	1	0.743	0.979	0.119
j	2	0.030	0.227	0.026
j	3	0.404	0.523	0.309

Patient Type Index		i = 1	i = 2	i = 3
j	1	50 min	60 min	10 min
j	2	10 min	20 min	10 min
j	3	30 min	40 min	20 min

Patient Type Indices	
Index	Type
1	follow-up
2	consult
3	2nd-opinion

Figure 15. Solution representation of patient types' inter-arrival time (C = 3, I = [10, 20, 30, 40, 50, 60]).



Step 1: Determine patients' sequence

	Patient 1	Patient 2	Patient 3	Patient 4	Patient 5	Patient 6	Patient 7	Patient 8
Patient Index	1	2	3	3	1	3	2	2
Patient Type	follow-up	consult	2nd-opinion	2nd-opinion	follow-up	2nd-opinion	consult	consult

+

Step 2: Determine patient types inter-arrival time

Patient Type Index		i = 1	i = 2	i = 3
j	1	50 min	60 min	10 min
j	2	10 min	20 min	10 min
j	3	30 min	40 min	20 min

Patient Type Indices	
Index	Type
1	follow-up
2	consult
3	2nd-opinion

Step 3: Generate the appointment schedule

	Patient 1	Patient 2	Patient 3	Patient 4	Patient 5	Patient 6	Patient 7	Patient 8
Current patient type (i)	1	2	3	3	1	3	2	2
Previous patient type (j)	N/A	1	2	3	3	1	3	2
Inter-arrival time (i,j)	N/A	10 min	40 min	20 min	10 min	30 min	10 min	20 min
Patient Type	follow-up	consult	2nd-opinion	2nd-opinion	follow-up	2nd-opinion	consult	consult
Arrival Time	8:00:00 AM	8:10:00 AM	8:50:00 AM	9:10:00 AM	9:20:00 AM	9:50:00 AM	10:00:00 AM	10:20:00 AM

Figure 16. Solution representation of patients' appointment schedule based on obtained results from steps 1 and 2.

patient arrival time. Equation (14) shows how the arrival time of a new patient (j) is calculated based on the arrival time of its previous patient (i), and their inter-arrival time. It is assumed that the first patient arrives at 8:00 am in the morning.

$$Arrival\ Time\ (j) = Arrival\ Time\ (i) + InterArrival\ Time\ (j, i) \tag{15}$$

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Mohammad Dehghanimohammadabadi https://orcid.org/0000-0002-4471-6049

References

1. Marynissen J and Demeulemeester E. Literature review on multi-appointment scheduling problems in hospitals. *Eur J Oper Res* 2019; 272: 407–419.
2. Ahmadi-Javid A, Jalali Z and Klassen KJ. Outpatient appointment systems in healthcare: a review of optimization studies. *Eur J Oper Res* 2017; 258: 3–34.
3. Kyambille GG and Kalegele K. *Enhancing patient appointments scheduling that uses mobile technology*. ArXiv Prepr. ArXiv160203337, 2016, https://arxiv.org/abs/1602 .03337
4. Gupta D and Denton B. Appointment scheduling in health care: challenges and opportunities. *IIE Trans* 2008; 40: 800–819.
5. Riley M. Operational dilemmas. *Int Hosp Ind*, 2012, p. 126. https://www.taylorfrancis.com/chapters/edit/10.4324/9780080509068-13/themes-prospects-bob-brotherton.
6. Carlo HJ and Martinez Y. Operational-level optimization of inbound intralogistics, 2016, https://digitalcommons.georgiasouthern.edu/cgi/viewcontent.cgi?article=1016&context=pmhr_2016
7. Robinson LW and Chen RR. Scheduling doctors' appointments: optimal and empirically-based heuristic policies. *Iie Trans* 2003; 35: 295–307.
8. Dehghanimohammadabadi, M. Date-Driven Simulation-Optimization (DSO): An Efficient Approach to Optimize Simulation Models with Databases. In Dorronsoro B, Pavone M, Nakib A, and Talbi E-G (eds.) Optimization and Learning, 5th International Conference, OLA 2022, Springer, Syracuse, Sicilia, Italy, 2022; J1684:109–26.
9. Denton B and Gupta D. A sequential bounding approach for optimal appointment scheduling. *IIE Trans* 2003; 35: 1003–1016.
10. Klassen KJ and Rohleder TR. Scheduling outpatient appointments in a dynamic environment. *J Oper Manag* 1996; 14: 83–101.
11. Klassen KJ and Rohleder TR. Outpatient appointment scheduling with urgent clients in a dynamic, multi-period environment. *Int J Serv Ind Manag* 2004; 15: 167–186.
12. Chien C-F, Tseng F-P and Chen C-H. An evolutionary approach to rehabilitation patient scheduling: a case study. *Eur J Oper Res* 2008; 189: 1234–1253.
13. Jerić SV and Figueira JR. Multi-objective scheduling and a resource allocation problem in hospitals. *J Sched* 2012; 15: 513–535.
14. Azadeh A, Baghersad M, Farahani MH, et al. Semi-online patient scheduling in pathology laboratories. *Artif Intell Med* 2015; 64: 217–226.
15. Rezaeiahari M and Khasawneh MT. An optimization model for scheduling patients in destination medical centers. *Oper Res Health Care* 2017; 15: 68–81.
16. Leeftink AG, Vliegen IMH and Hans EW. Stochastic integer programming for multi-disciplinary outpatient clinic planning. *Health Care Manag Sci* 2019; 22: 53–67.
17. Klassen KJ and Yoogalingam R. Appointment scheduling in multi-stage outpatient clinics. *Health Care Manag Sci* 2019; 22: 229–244.
18. Harper PR and Gamlin HM. Reduced outpatient waiting times with improved appointment scheduling: a simulation modelling approach. *Spectr* 2003; 25: 207–222.
19. White DL, Froehle CM and Klassen KJ. The effect of integrated scheduling and capacity policies on clinical efficiency. *Prod Oper Manag* 2011; 20: 442–455.
20. Romero HL, Dellaert NP, Van Der Geer S, et al. Admission and capacity planning for the implementation of one-stop-shop in skin cancer treatment using simulation-based optimization. *Health Care Manag Sci* 2013; 16: 75–86.
21. Pérez E, Ntaimo L, Malavé CO, et al. Stochastic online appointment scheduling of multi-step sequential procedures in nuclear medicine. *Health Care Manag Sci* 2013; 16: 281–299.
22. Kuiper A and Mandjes M. Appointment scheduling in tandem-type service systems. *Omega* 2015; 57: 145–156.
23. Guo M, Wagner M and West C. Outpatient clinic scheduling: a simulation approach. In: *Proceedings of the 36th conference on Winter simulation*, 2004, pp. 1981–1987, https://ieeexplore.ieee.org/document/1371559
24. Ogulata SN, Cetik MO, Koyuncu E, et al. A simulation approach for scheduling patients in the department of radiation oncology. *J Med Syst* 2009; 33: 233–239.
25. Scala P, Mota MM, Wu C-L, et al. An optimization–simulation closed-loop feedback framework for modeling the airport capacity management problem under uncertainty. *Transp Res Part C Emerg Technol* 2021; 124: 102937.
26. Klassen KJ and Yoogalingam R. Improving performance in outpatient appointment services with a simulation optimization approach. *Prod Oper Manag* 2009; 18: 447–458.
27. Klassen KJ and Yoogalingam R. Strategies for appointment policy design with patient unpunctuality. *Decis Sci* 2014; 45: 881–911.
28. Dehghanimohammadabadi M, Rezaeiahari M and Keyser TK. Simheuristic of patient scheduling using a table-experiment approach: Simio and MATLAB integration application. In: *Proceedings of the 2017 Winter simulation conference*, 2017, p. 238, https://ieeexplore.ieee.org/document/8248015
29. Saremi A, Jula P, ElMekkawy T, et al. Appointment scheduling of outpatient surgical services in a multi-stage operating room department. *Int J Prod Econ* 2013; 141: 646–658.
30. Saremi A, Jula P, ElMekkawy T, et al. Bi-criteria appointment scheduling of patients with heterogeneous service sequences. *Expert Syst Appl* 2015; 42: 4029–4041.
31. Baesler FF and Sepúlveda JA. Multi-objective simulation optimization for a cancer treatment center. In: *Proceeding of the 2001 Winter simulation conference (Cat. No.01CH37304)*, 2001, vol. 2, pp. 1405–1411, https://ieeexplore.ieee.org/document/977463
32. Rezaeiahari M and Khasawneh MT. Simulation optimization approach for patient scheduling at destination medical centers. *Expert Syst Appl* 2020; 140: 112881.
33. Comis M, Cleophas C and Büsing C. Patients, primary care, and policy: agent-based simulation modeling for health care decision support. *Health Care Manag Sci* 2021; 24: 799–826.
34. Xie Z and Wang C. A periodic repair algorithm for dynamic scheduling in home health care using agent-based model. In: *2017 IEEE 21st international conference on computer supported cooperative work in design (CSCWD)*, 2017, pp. 245–250, https://ieeexplore.ieee.org/document/8066702

35. Mohammadi I, Wu H, Turkcan A, et al. Data analytics and modeling for appointment no-show in community health centers. *J Prim Care Community Health* 2018; 9: 2150132718811692.
36. Marynissen J and Demeulemeester E. Literature review on multi-appointment scheduling problems in hospitals. *Eur J Oper Res* 2019; 272: 407–419.
37. Matulis JC and McCoy R. Patient-centered appointment scheduling: a call for autonomy, continuity, and creativity. *J Gen Intern Med* 2021; 36: 511–514.
38. Shao L. A survey of beam intensity optimization in IMRT. In: *Proceedings of the 40th annual conference of the Operational Research Society of New Zealand*, Wellington, 2005, pp. 255–264, https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.472.1238&rep=rep1&type=pdf
39. Casjens S, Schwender H, Brüning T, et al. A novel crossover operator based on variable importance for evolutionary multi-objective optimization with tree representation. *J Heuristics* 2015; 21: 1–24.
40. Olympio KR and Gandhi F. Optimal cellular core topologies for one-dimensional morphing aircraft structures. *J Mech Des* 2012; 134: 081005.
41. Juan AA, Faulin J, Grasman SE, et al. A review of simheuristics: extending metaheuristics to deal with stochastic combinatorial optimization problems. *Oper Res Perspect* 2015; 2: 62–72.
42. Rabbani M, Mousavi Z and Farrokhi-Asl H. Multi-objective metaheuristics for solving a type II robotic mixed-model assembly line balancing problem. *J Ind Prod Eng* 2016; 33: 472–484.
43. Guan H. *MOBA: a multi-objective bounded-abstention model for two-class cost-sensitive problems*. ArXiv Prepr. ArXiv190507297, 2019, https://arxiv.org/abs/1905.07297
44. Coello CC and Lechuga MS. MOPSO: a proposal for multiple objective particle swarm optimization. In: *Proceedings of the 2002 congress on evolutionary computation: CEC'02* (Cat. No.02TH8600), 2002, vol. 2, pp. 1051–1056, https://ieeexplore.ieee.org/document/1004388/
45. Peimankar A, Weddell SJ, Jalal T, et al. Evolutionary multi-objective fault diagnosis of power transformers. *Swarm Evol Comput* 2017; 36: 62–75.
46. Liu Y, Liao X and Zhang R. An enhanced MOPSO algorithm for energy-efficient single-machine production scheduling. *Sustainability* 2019; 11: 5381.
47. Abdullah M, Al-Muta'a EA and Al-Sanabani M. Integrated MOPSO algorithms for task scheduling in cloud computing. *J Intell Fuzzy Syst* 2019; 36: 1823–1836.
48. Sheikhalishahi M, Eskandari N, Mashayekhi A, et al. Multi-objective open shop scheduling by considering human error and preventive maintenance. *Appl Math Model* 2019; 67: 573–587.
49. Ershadi MM, Ershadi MJ and Niaki STA. An integrated HFMEA-DES model for performance improvement of general hospitals. *Int J Qual Reliab Manag* 2021; 38: 1–24.
50. Ahmadi E, Garcia-Arce A, Masel DT, et al. A metaheuristic-based stacking model for predicting the risk of patient no-show and late cancellation for neurology appointments. *IISE Trans Healthc Syst Eng* 2019; 9: 272–291.
51. Adeli M, Zandieh M and Motameni A. Integrated sourcing and inventory decisions considering sources' disruptions with a hybrid simulation-MOPSO-TOPSIS approach: a pharmaceutical case study. *J Ind Eng Manag* 2019; 6: 103–119.
52. Arisha A and Rashwan W. Modeling of healthcare systems: past, current and future trends. In: *Proceedings of the 2016 Winter simulation conference (WSC)*, 2016, pp. 1523–1534, https://www.anylogic.fr/upload/iblock/d99/d99c703892684ade9723f7f09d0512ee.pdf
53. Dehghanimohammadabadi M and Keyser TK. Intelligent simulation: integration of SIMIO and MATLAB to deploy decision support systems to simulation environment. *Simul Model Pract Theory* 2017; 71: 45–60.
54. Hamrock E, Paige K, Parks J, et al. Discrete event simulation for healthcare organizations: a tool for decision making. *J Healthc Manag* 2013; 58: 110–124.
55. Fan X, Tang J, Yan C, et al. Outpatient appointment scheduling problem considering patient selection behavior: data modeling and simulation optimization. *J Comb Optim* 2021; 42: 677–699.
56. Roberts SD. Tutorial on the simulation of healthcare systems. In: *Proceedings of the Winter simulation conference*, 2011, pp. 1408–1419, https://dl.acm.org/doi/abs/10.5555/2431518.2431686
57. Dehghanimohammadabadi M, Rezaeiahari M and Keyser TK. Simheuristic of patient scheduling using a table-experiment approach—Simio and MATLAB integration app lication, 2017, https://ieeexplore.ieee.org/document/8248015
58. Dehghanimohammadabadi M, Keyser TK and Cheraghi SH. A novel Iterative Optimization-based Simulation (IOS) framework: an effective tool to optimize system's performance. *Comput Ind Eng* 2017; 111: 1–17.
59. Dehghanimohammadabadi M. Iterative optimization-based simulation (IOS) with predictable and unpredictable trigger events in simulated time. *Western New England University*, 2016, https://www.proquest.com/docview/1773511260?pq-origsite=gscholar&fromopenview=true (accessed 30 May 2016).
60. National Breast Cancer Foundation Inc., 2018, https://www.nationalbreastcancer.org/breast-cancer-facts (accessed 20 August 2010).
61. Potter S, Govindarajulu S, Shere M, et al. Referral patterns, cancer diagnoses, and waiting times after introduction of two week wait rule for breast cancer: prospective cohort study. *Br Med J* 2007; 335: 288.
62. Britton P, Duffy SW, Sinnatamby R, et al. One-stop diagnostic breast clinics: how often are breast cancers missed. *Br J Cancer* 2009; 100: 1873–1878.
63. Shaaban S and Hudson S. The performance of unpaced serial production lines with unequal coefficients of variation of processing times. *Int J Comput Appl Technol* 2009; 34: 122–128.
64. Romero-Silva R, Marsillac E, Shaaban S, et al. Serial production line performance under random variation: dealing with the ''law of variability.'' *J Manuf Syst* 2019; 50: 278–289.
65. Jáuregui GRR, Pérez AKG, González SH, et al. Analysis of the emergency service applying the queueing theory. *Contad Adm* 2017; 62: 733–745.
66. Altiok T and Melamed B. *Simulation modeling and analysis with Arena*. Amsterdam: Elsevier, 2010.

67. Gul S, Denton BT, Fowler JW, et al. Bi-criteria scheduling of surgical services for an outpatient procedure center. *Prod Oper Manag* 2011; 20: 406–417.
68. Zhang Z, Berg BP, Denton BT, et al. Appointment scheduling and the effects of customer congestion on service. *IISE Trans* 2019; 51: 1075–1090.
69. Deb K, Pratap A, Agarwal S, et al. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans Evol Comput* 2002; 6: 182–197.
70. Zitzler E, Laumanns M and Thiele L. *SPEA2: improving the strength Pareto evolutionary algorithm*. TIK-Report 103, 2001, https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.28.7571&rep=rep1&type=pdf
71. Zitzler E and Künzli S. Indicator-based selection in multiobjective search. In: *International conference on parallel problem solving from nature*, 2004, pp. 832–842, https://link.springer.com/chapter/10.1007/978-3-540-30217-9_84

Author biographies

Mohammad Dehghanimohammadabadi is Associate Teaching Professor of Mechanical and Industrial Engineering, Northeastern University, Boston, MA, USA. His research is mainly focused on developing and generalizing simulation and optimization frameworks in different disciplines. This article is part of his initiatives to develop a framework to integrate Discrete Event Simulation with multi-objective optimization models.

Mandana Rezaeiahari, PhD, is an Assistant Professor of Health Policy and Management at the University of Arkansas for Medical Sciences (UAMS). Her research interest is in predictive analytics, simulation, and optimization, and she has relevant research experiences in the application areas of healthcare operations management and health disparities.

Javad Seif, PhD, is an Assistant Professor of Industrial and Manufacturing Engineering at California State Polytechnic University, Pomona (Cal Poly Pomona). He teaches various operations research courses to undergraduate and graduate students, as well as quality and reliability engineering courses. His research interest is in operations and maintenance scheduling, health monitoring and reliability analysis, and various applications of mathematical optimization.